Journal of Quantitative Finance and Economics
Volume 5; Number 2; 2023; pp : 259-273
https://DOI:10.47509/JQFE.2023.v05i02.04

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Comparative Analysis of Error Innovation Distributions in Modelling Volatility of Nigeria Stock Exchange

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To cite this paper:

OLUBIYI Adenike Oluwafunmilola, OLAYEMI Michael Sunday, ADIGUN Kehinde Abimbola, & OLAJIDE Oluwamayowa Opeyimika (2023). Comparative Analysis of Error Innovation Distributions in Modelling Volatility of Nigeria Stock Exchange. *Journal of Quantitative Finance and Economics.* 5(2), 259-273. https://DOI:10.47509/JQFE.2023.v05i02.04

Abstract: *In the estimation of volatility models, this paper compares the new distribution of innovation error. An empirical database of the daily returns of the Nigeria Stock Exchange (NSE) index from 2012 to 2022 was used to compare the standardized exponentiated Gumbel error innovation distribution (SEGEID) with the existing error distribution. The data are stationary without transformation, according to the statistics of stationary, but there is heteroscedasticity, according to the statistics of the ARCH effect using the statistics of ADF. With a probability value of 0.00 in both the new error distribution and the current distribution, the estimates of the volatility model are significant. The results showed that GARCH (1,1) with a SEGEID error distribution surpassed other model with lower AIC values. In the study simulation, GARCH (1,1) with SEGEID was more effective than other error distributions and showed the effectiveness and effectiveness of SEGEID.*

Keywords: *Volatility models, Simulation, Standardized exponentiated, stocks and error distributions*

Jel Classification: *C58, F47, C59*

I. Introduction

"Engle (1982) suggested using the normal distribution to estimate the errors of his proposed volatility model, since the error distribution is one of the most important techniques used to estimate the parameters of any volatility model. In volatility model estimation, this error distribution has made more progress than the student distribution. Bollerslev (1987) stated in a paper that six types of error distributions are preferred in volatility modelling

Article Info.: Received: 22 October 2023; Revised: 19 November 2023; Accepted: 26 November 2023; Online: 30 December 2023

because they play an important role in predicting the parameters of heterogeneous models (Su, 2010). (Bali2007; Shamiri 2009). Among them are normal distributions, skew normal distributions, student t distributions, student t distributions with skews, general error distributions and general error distributions with skews. Olayemi and Olubiyi (2022) proposed a new error distribution from a standardized exponential gumbel error innovation distribution (SEGEID), which would model some volatility models, estimate parameters using the proposed error distribution (SEGEID), and compare it to existing error distributions, particularly due to the limitations of existing error innovation distributions, which are difficult to capture extreme values, heavy tails, etc. In this study, we investigate the importance of additional parameters for the error innovation distribution (SEGEID) in terms of providing the best fit compared to other error innovation distributions".

II. Review of Literature

When Gauss first introduced normal distributions in 1809 and used the min 1816, it was called normal distributions, later renamed Gaussian distributions in his honour. "The normal curve on which Gauss distribution is based was first presented by Abraham de Moivre in the 18th century. In 1982, Engle proposed a distribution of innovations for estimating volatility models, which was used to evaluate the ARCH model. Bollerslev (1986) chose this normal distribution as an error innovation in the GARCH model and used it to estimate volatility models. However, many studies of financial time series show that they show features that do not account for normal innovation distribution of errors, such as fat tails, clusters, and leverage effects. Consequently, more precise distributions of errors in the innovation division have been developed to predict volatility in financial data. Student T distribution was originally identified by William (1908) as symmetric and bell-shaped like normal distribution with strong tail. The Student T distribution is a subset of the normal distribution with increasing degrees of freedom". "The student-t distribution was used to estimate Bollerslev's volatility models (1987). The error innovation of this distribution was used to capture the constraints of normal distributions. Bollerslev (1987) demonstrated that the student t distribution perfectly captures the observed Kurtosis".

"To model financial time series, such as those with exchange rates and stock returns, he pioneered the use of GARCH modelling and pioneered the error innovation of the student t distribution. In estimating the EGARCH model, Nelson (1991) advocated the use of general error distribution (GED)to innovate errors. The GED includes representations for normal distributions, Laplace and uniform distributions. According to Nelson (1991), GED is more attractive than the distribution of student-t errors for the achievement of stability. Another distribution is contrary to the normal distribution of student-t". O'Hagan and Leonard (1976) developed a skewed normal distribution to explain the asymmetry of the normal distribution. "Azzalini's contribution to distribution analysis was more detailed (1985, 1986, 2005). Fernandez and Steel (1998) used this distribution to introduce errors into the volatility model. Discourse normal distribution is an alternative to normal distribution and explains the asymmetric nature of error innovation distribution for asset return. It is essentially an extended version of the standard distribution. In particular, skewed normal distribution is characterized by the incorporation of skewness parameters. Hansen introduced the distribution of non-normal errors known as Student t"(1994).

"Due toits symmetry, the student-t distribution can only model symmetrical returns, not asymmetrical returns. The introduction of distribution helped overcome this restriction. Hansen (1994) reduced the spread of student-t distributions by incorporating a differential parameter to create a differential student-t distribution. Jones (2001) and Jones and Faddy (2003) proposed new generalizations of student-t distributions of a single variable. In 1994, Hansen proposed this distribution to simulate financial returns using a volatility model. The generalized error distribution Skewed was developed in 1998by Theodossiou and is a variant of the Laplace error distribution Skewed for error innovations. In order to explain the skewness of the general error distribution (GED) and the skewness of the Laplace distribution, which is a special example of the general distribution, Theodossiou introduced a skew parameter. This was the first time GED asymmetry was exploited to create new error types. In 2022, Olayemi and Olubiyi proposed a new error innovation distribution called standardized exponentiated Gumbel error innovation distribution (SEGEID) in a model volatility model, which captures some of the limitations of the six existing main distributions"

III. Methodology

"Computation of return series for price"

Let

$$r_{sk} = log\left(\frac{y_t}{y_{t-1}}\right) t = 1,2, \dots, n \quad (1)$$

"Where y_t and y_{t-1} are the present and previous closing prices at time t and r_{sk} is the returns series".

Stock Market Volatility

"Generally, in financial market, volatility is often known as standard deviationσ or variance σ^2. A stock's volatility is a gauge of the degree of uncertainty surrounding the returns it will produce. The parameter is often generated using a number of data from the empirical sample in the manner shown below"

$$\sigma^2 = \frac{1}{n-1}\sum_{t=1}^{n}(r_t - \mu)^2 \tag{2}$$

"Where μ is the mean return and $r_{sk} = log\left(\frac{p_t}{p_{t-1}}\right)$ "

Computation of the Conditional Error Term

"The Error (ε_t) term is computed as:"

$$\varepsilon_t = r_t - \mu \tag{3}$$

"Where r_{sk} is the return of the series and μ is the mean of the series. For single observation return series, the error term is given as:"

$$\varepsilon_i = r_i - \mu \tag{4}$$

"Where is the individual error term, is the individual return series and is the grand mean of the whole return series"

Computation of the Variance Term

"The unconditional variance computation formula is given as:"

$$\sigma_t^2 = var(r_{sk}) \tag{5}$$

"Where is the return of the series. For single observation return series, the variance is given as:"

$$\sigma_{t-1}^2 = var(r_1, r_2) \tag{6}$$

Volatility Models

Autoregressive Conditional Heteroscedasticity (Arch) Model

"Engle (1982) proposed the ARCH (q) model which formulates volatility model as follows:"

$$\sigma_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \cdots + \alpha_q\varepsilon_{t-q}^2 \tag{7}$$

This can also be express as:

$$\sigma_t^2 = \alpha_0 + \sum_{i}^{q} \alpha_i \varepsilon_{t-i}^2 \tag{8}$$

"Where the parameters of the model and q is the order of ARCH terms"

Generalize Autoregressive Conditional Heteroscedasticity (GARCH) Model

"The GARCH (p, q) model proposed by Bollerslev (1986) formulates volatility as follows:"

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_q \varepsilon_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \cdots + \beta_p \sigma_{t-p}^2 \tag{9a}$$

"Alternatively, it can be stated as:"

$$\sigma_t^2 = \alpha_0 + \sum_{i}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j}^{p} \beta_j \sigma_{t-j}^2 \tag{9b}$$

"Where$\alpha_0 > 0, \alpha_i \geq 0, \beta_j \geq 0$ and $\alpha_i + \beta_j < 1$ for all i and j while q is the ARCH order terms, and p is the GARCH order terms"

Standardized Normal Error Innovation Distribution

"The following is the standardized normal error innovation distribution that Engle (1982) proposed"

$$f(z_t) = \frac{1}{\sqrt{2\pi}} e^{-z^{21}/_2} \qquad -\infty < z_t < \infty \tag{10}$$

Skewed Normal Error Innovation Distribution

"Azzalini made the initial skew extension of the regular error innovation distribution (1985). The following is the skewed-normal (SN) distribution's probability density function (pdf):

$$\phi(z;\lambda) = 2\phi(z)\Phi(z\lambda); z \in \Re, \lambda \in \Re \tag{11}$$

Where λ is a second parameter that regulates skewness, ϕ is the normal distribution's pdf, and Φ is its cumulative distribution function (cdf). The Skewed Normal distribution, which is utilised for error innovation and is provided by, was proposed by Fernandez and Steel in 1998"

$$f(z) = \frac{1}{w\pi} e^{-\frac{(z-\eta)^2}{2w^2}} \int_{-\infty}^{\vartheta \frac{z-\eta}{w}} e^{\frac{-1^2}{2}} dt, -\infty < z < \infty, -\infty < z_t < \tag{12}$$

Where η is the location, w is the scale and ϑ denotes the shape parameter"

Generalized Error Innovation Distribution

Generalized Error Innovation Distribution was proposed by Nelson (1991) and is given by;

$$f(z_t) = \frac{u\ exp(-0.5|z_t|\lambda)^u}{2^{\left(1+\frac{1}{u}\right)}\Gamma(U^{-u})\lambda} \quad U > 0 \tag{13}$$

"Where U is the shape parameter" and

$$\lambda = \left\{ 2^{\langle\frac{-2}{u}\rangle} \frac{\Gamma\left(\frac{1}{u}\right)}{\Gamma\left(\frac{3}{u}\right)} \right\}^{\frac{1}{2}}$$

Student T Error Innovation Distribution

"Bollerslev (1987) first developed the standardized student-t error innovation distribution, which is represented as"

$$f(z_t|u) = \frac{\Gamma\left(\frac{u+1}{2}\right)}{\sqrt{(u-2)\pi}\Gamma\left(\frac{u}{2}\right)}\left(1 + \frac{z_t^2}{u-2}\right)^{-\frac{u+1}{2}} \tag{14}$$

Standardized Skewed Student-t Error Innovation Distribution

"Hansen (1994) first proposed the standardised skewed student t error innovation distribution, which is represented by:"

$$f(z; \mu, \sigma, u, \lambda) = \begin{cases} bc\left(1 + \dfrac{1}{u-2}\left(\dfrac{b\left(\frac{z-\mu}{\sigma}\right) + a}{1-\lambda}\right)^2\right)^{-\frac{u+1}{2}}, & if\ z < -\dfrac{a}{b} \\ bc\left(1 + \dfrac{1}{u-2}\left(\dfrac{b\left(\frac{z-\mu}{\sigma}\right) + a}{1-\lambda}\right)^2\right)^{-\frac{u+1}{2}}, & if\ z \geq -\dfrac{a}{b} \end{cases} \tag{15}$$

"Where u is the shape parameter with $2<u<\infty$ and λ is the skewness parameter"

$$a = 4\lambda c\left(\frac{u-2}{u-1}\right), b = 1 + 3\lambda^2 - a^2, \qquad c = \frac{\Gamma\left(\frac{u+1}{2}\right)}{\sqrt{\pi(u-2)\Gamma\left(\frac{u}{2}\right)}}$$

Standardized Skewed Generalized Innovation Distribution

"Theodossiou (1998) proposed the Standardized Skewed Generalized Innovation Distribution, which is represented as follows:"

$$f(z_t|u,\varepsilon,\theta,\delta) = \frac{u}{2\theta\Gamma\left(\frac{1}{u}\right)} exp\left[-\frac{|z_t-\delta|^u}{[1+sign(z_t-\delta)\varepsilon]^u\theta^u}\right] \tag{16}$$

$$\theta > 0, -\infty < z_t < \infty, u > 0, -1 < \varepsilon < 1$$

Where

$$\theta = \Gamma\left(\frac{1}{u}\right)^{0.5}\Gamma\left(\frac{3}{u}\right)^{-0.5} S(\varepsilon)^{-1},$$

$$\delta = 2\varepsilon S(\varepsilon)^{-1},$$

$$S(\varepsilon) = \sqrt{1 + 3\varepsilon^2 - 4A^2\varepsilon^2},$$

$$A = \Gamma\left(\frac{2}{u}\right)\Gamma\left(\frac{1}{u}\right)^{-0.5}\Gamma\left(\frac{3}{u}\right)^{-0.5}$$

"Where $u > 0$ is the shape parameter, ε is a Skewness with $-1 < \varepsilon < 1$

Standardized Exponentiated Gumbel Error Innovation Distribution (Segeid)"

$$g(\varepsilon_t;\alpha,\sigma_t) = \frac{\alpha}{\left(\sigma_t^2\right)^{\frac{1}{2}}}\left[1-\exp\left\{-\exp\left(\frac{\varepsilon_t}{\sigma_t^2}\right)\right\}\right]^{\alpha-1}\exp\left(\frac{\varepsilon_t}{\sigma_t^2}\right)\exp\left\{-\exp\left(\frac{\varepsilon_t}{\sigma_t^2}\right)\right\}\left(\frac{1}{\left(\sigma_t^2\right)^{\frac{1}{2}}}\right) \tag{17}$$

α is the shape parameter, σ_t is the volatility models with vector parameters"

IV. Results and Discussion

Empirical Result

"The empirical analysis of the returns of NSE indexes was carried out in a series. The results obtained, as shown in Table 1, show that the mean return series is positive, positive, and the index return is strongly skewed. The results of the Jarque-Bera statistics revealed that the NSE index return series is normally not distributed, since the p value is less than1%"

Table 1: Descriptive Statistic

Statistic	*Returns of NSE Index*
Mean	1.000033
Standard Deviation	0.000945
Skew.	0.336632
Kurt.	8.481275
Jarque-Bera	3302.643
P-Value	0.000
Observations	2599

Normality Test

The results of the normalization test for the return of the NSE are shown in table 2. Analysis using Kolmogorov-Smirnov (K-S) and Shapiro-Wilk (S-W) statistics showed that returns to the NSE stock were normally not distributed because P values were less than 0.01.

Table 2: Test of Normality of the Return of Nigeria Stock Exchange (NSE) Index Return

Kolmogorov-Smirnov (K-S)				*Shapiro-Wilk (S-W)*		
Statistics	*Df*	*p-value*		*Statistic*	*Df*	*P-Value*
NSE	0.385	3198	0.000	0.051	3189	0.000

Stationarity Test

"By observing the time models of the series, researchers were able to investigate the stationarity of the return series. The NSE price and yield series is unchanged, as shown in Figure 1. The formal stationarity test using the Dickey-Fuller (ADF) enhanced test was also carried out. "The results show that all Dickey-Fuller augmented test statistics are smaller than their critical values of 0.01, as shown in table 3 and, as a result, there is no unit root and no conversion requirement because all returns are stationary"

Table 3: Augmented Dickey-Fuller (ADF) Test of Stationarity of the All Share Index Return Series Nigeria Stock Exchange

Stocks		*ADF Test Statistics*	*Comment*
NSE Index Returns	Intercept	-24.62147	"Stationary at stated level without transformation"
	Trend and intercept	-25.98503	"Stationary at stated level without transformation"

1% critical = -3.342675

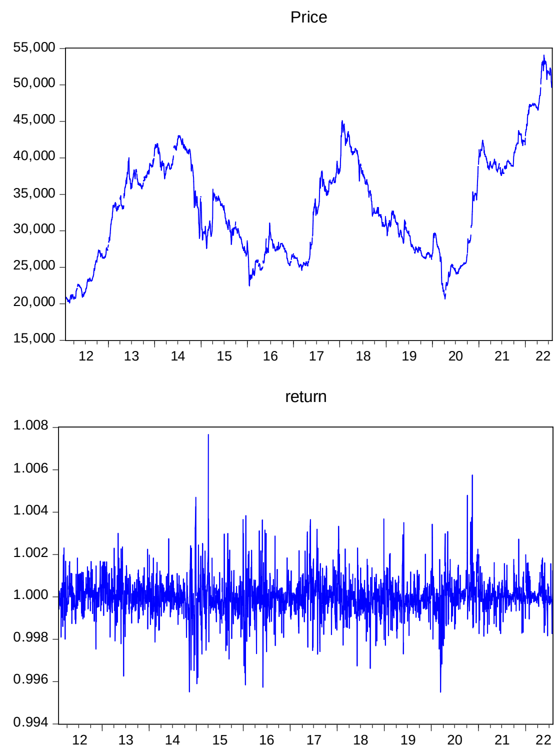

Figure 1: volatility plot of both Price and Returns of NSE Index Returns

ARCH Effect Test

"The method of Lagrange Multiplier (LM)is tested. Table 4.4shows the results of the P-value and F statistics achieved at various delays. For NSE stock returns, the F statistics value is 1 per cent significant. Consequently, NSE index returns meet the requirements of heteroscedastic model evidence of the presence of the ARCH effect"

Table 4: Lagrange Multiplier Test of the Presence of ARCH Effect

	ARCH Effect	*F-Statistic*	*P-value*
NSE Index Returns	At lag 1-4	768.44	0.000
	At lag 1-6	443.82	0.000
	At lag 1-10	270.92	0.000

Estimates of the Parameters of GARCH Family Models based on Nigeria Stock Exchange (NSE) Index Returns

"The GARCH model parameter estimates are presented in Table 5, and the proposed error distribution is calculated using the maximum likelihood estimate of the NSE index return data. The parameter estimates cover the normal, student-t, generalized, irregular, irregular and irregular distributions of generalized error. In all models, the coefficient β_1 is statistically significant (which affects persistence). Most of the models taken into account have significant GARCH models ($p<0.05$ and $p<0.01$), which indicates that significant changes usually cause large fluctuations, while small changes usually cause small fluctuations. In the GARCH models taken into account for this analysis at different error distributions, the leverage effect was significant ($p<0.05$) to assess whether there is a negative link between asset returns and fluctuations"

Table 5: Estimates of the Parameters of GARCH Models on Nigeria Stock Exchange (NSE) Index Returns Using the Six Existing and SEGEID

Model	*Error*	*w*	α_1	β_1	γ_1	*Shape*
GARCH (1,1)	NORM	6.574×10^{-10}	$2.748 \times 10^{-01***}$	$1.00 \times 10^{-08***}$		2.000^{**}
	STD-T	$8.775 \times 10^{-03*}$	$1.842 \times 10^{-01***}$	$7.764 \times 10^{-01***}$		
	GED	$3.091 \times 10^{-06***}$	$1.369 \times 10^{-01***}$	$9.524 \times 10^{-01***}$		1.163^{***}
	SNORM	6.574×10^{-10}	$3.649 \times 10^{-01***}$	$1.00 \times 10^{-08***}$		2.000^{*}
	SSTD-T	$8.875 \times 10^{-03*}$	$1.852 \times 10^{-01**}$	$5.764 \times 10^{-01***}$		
	SGED	$3.081 \times 10^{-06***}$	$1.258 \times 10^{-01***}$	$7.524 \times 10^{-01***}$		1.163^{***}
	SEGEID	0.15676	-0.23615	1.11573^{*}		7.5543

* at 5%, ** at 1% and *** at 10% significant

Comparison of Error Innovation Distributions for Fitness and Model Selection of Some GARCH Family Models on Nigeria Stock Exchange (NSE) Index Returns

"The results of fitness tests and model selection using likelihood functions and Akaike Information Criteria (AICs) are shown in table 7. According to its largest log probability and lowest Akaike information criteria (AIC), SEGEID was considered to be the best GARCH model to be studied, with SEGEID being better than other error distributions based on the assessment of the entire error distribution"

Table 7: Comparison of Error Innovation Distribution for Model Selection of Some GARCH Family Models on Nigeria Stock Exchange (NSE) Index Returns

Models	*Error Distributions*	*LL*	*AIC*
GARCH (1,1)	NORMAL	12848.89	-8.9220
	STUDENT-T	5830.812	-9.3418
	GENERALIZED	8476.708	-7.0365
	SNORMAL	9567.564	-11.9304
	SSTUDENT-T	100000.546	-9.0206
	SGENERALIZED	987.675	-8.0903
	SEGEID	444000.697	**-15.0394**

Simulation Study: Logistic Distribution

Comparison of Error Innovation Distribution for Forecasting Performance of Estimated GARCH Model on NSE Index Returns

"Table 8 shows the predictive performance of estimates models in different error distributions, using Root Mean Square Errors in current and new error innovation distributions (RMSE). The lowest-root average square error (RMSE) model is considered to be the best model for performance prediction determined by different error innovation distributions. The results show that the SEG error innovation distribution of GARCH (1,1) compared toother error innovation distributions in forecast performance taken into account in this paper"

Table 8: Comparison of Error Innovation Distribution for Forecasting Evaluation of Some GARCH Family Models Based on NSE Index Returns

Model	*Error distributions*	*RMSE*
GARCH (1,1)	NORM	0.0710
	STD-T	5.5754
	GED	5.7548
	SNORM	0.4315
	SSTD-T	0.0039
	SGED	0.1430
	SEGEID	0.0001

RMSE- ROOT MEAN SQUARE ERROR, BOLDED VALUES ARE THE LEAST ROOT MEAN SQUARE ERROR (RMSE)

Simulation Study: Logistic Distribution

"First, we collected performance information on each stock index for specific financial investments, using MTN Nigeria Communications Ltd.in this study. INDEX, a separate dataset of (NSE index returns). The Easy Fit program is used to estimate all continuous distributions with domain boundaries from negative to positive infinity. Based on parameter estimates, the best adaptable distribution is selected for simulation, and in this case, the best adaptable logistic distribution for a given data set is chosen. Table 8 shows the results of the Good Adjustment Test for Continuous Distributions. Chi-square, Anderson Darling, and Kolmogorov-Smirnov have rated the distribution's fit test. Anderson Darling, however, is more effective than Kolmogorov Smirnov in large sample sizes, and chi squares and Kolmogorov Smirnov are also effective in non-parametric tests (Nornadiah and Yap, 2011). Consequently, 900returndata were simulated using a logistic distribution"

Table 9: Goodness of Fit Summary

	Distribution	*Kolmogorov Smirnov*		*Anderson Darling*		*Chi-Square*	
		Statistic	*Rank*	*Statistic*	*Rank*	*Statistic*	*Rank*
1	Error	0.28646	4	92.022	2	2289.6	1
2	Error Function	1	9	N/A	N/A		
3	Gumbel Max	0.43748	8	139.96	7	2592.7	7
4	Gumbel Min	0.43742	7	155.74	8	N/A	
5	Hyper secant	0.27829	2	100.98	4	2344.7	4
6	Johnson SU	0.29569	6	106.15	5	2341.8	3
7	Laplace	0.28656	5	92.033	2	2289.6	2
8	Logistic	0.27697	1	88.52	1	2411.1	5
9	Normal	0.27853	3	116.86	6	2514.2	6
10	Cauchy	No fit					
11	Student's t	No fit					

Simulation Results Comparison for Fitness and Model Selection of Some GARCH Family Models

"Table 10 shows the results of the fitness and model selection based on the Akaike information criteria and the likelihood function, respectively (AIC). In GARCH (1,1), we examined the distribution of errors in innovations. According to this result, SEGEID's GARCH (1,1) exceeded other error distributions because of the minimum AIC value. This is demonstrated by model and error distribution"

Table 10: "Simulation Result on error distribution of Some GARCH Family Models"

Models	*Error Distributions*	*LL*	*AIC*
GARCH (1,1)	NORM	145393.289	-9.9220
	STD-T	15456.812	-7.5418
	GED	14576.708	-6.0365
	SNORM	19567.564	-11.9304
	SSTD-T	19100.546	-10.2206
	SGED	147800.675	-9.0903
	SEGEID	**1944530.899**	**-15.0394**

Simulation Results Forecasting Performance of Estimated GARCH Family Models

"Table 11 shows the simulation results of model forecast performance estimates for different error distributions of existing and new error innovation distributions (RMSEs). The least root mean square error (RMSE)model is considered to be the best model for predicting performance, depending on the different distribution of error innovation. The results show that SEG error innovation distributions in GARCH (1,1) outperform other error innovation distributions in forecast performance taken into account in this study"

Table 11: Simulation on forecasting performance of Some GARCH Family Models

Model	*Error distributions*	*RMSE*
GARCH (1,1)	NORM	2.0710
	STD-T	1.5554
	GED	4.4548
	SNORM	1.3315
	SSTD-T	0.00139
	SGED	0.00430
	SEGEID	0.00004

RMSE- ROOT MEAN SQUARE ERROR, BOLDED VALUES ARE THE LEAST ROOT MEAN SQUARE ERROR (RMSE)

Discussion of Findings

"This study found that while the current six error innovation distributions calculating GARCH model parameters in terms of fitness and prediction performance are better, a new error innovation distribution proposed by Olayemi and Olubiyi (2022) can increase the flexibility of existing distributions. This was achieved by adopting the Exponentiated Gumbel distribution. As a result, a standardized index Gumbel error innovation distribution has been proposed and evaluated based on empirical research using the stock index returns of the Nigeria Stock Exchange (NSE). According to the findings of the NSE index returns, standardized indices gumbel distribution, a new error innovation distribution (SEGEID), provided the lowest AIC and root mean square error. This indicates that the proposed distribution is superior to the existing six (6) error innovation distributions in terms of prediction performance when estimating GARCH parameters (1,1). These results indicate that SEGEID's proposed distributions were superior to the other six distributions studied (NORM, STD, GED, SNORM, SSTD, and SGED).

Furthermore, when comparing the estimated parameters of GARCH (1,1) to six other existing error innovation distributions, the Standardized Exponentiated Gumbel distribution (SEGEID) provides the lowest AIC and root average square error. This indicates that the proposed distribution is better for health and forecasting performance. This result means that SEGEID is better than the other six distributions studied in the study (NORM, STD, GED, SNORM, SSTD, and SGED), compared to the other six distributions of error innovation (Segregated Exponentiated Gumbel Distribution, New Error Innovation Distribution (SEGEID), which gives the lowest AIC and root mean square errors. The results show that SEGEID's proposed distribution is better than the six current distributions in the study (NORM, STD, GED, SNORM, SSTD, and SGED)"

Conclusion

"In order to create more flexible distributions of error innovation, the GARCH (1) model family was used in this study. The recommended error distribution is an improvement over the current error innovation distribution, and has shown that it exceeds other existing distributions in terms of prediction accuracy and suitability using index return and simulation return data. Both scientists and investors can learn a lot from this discovery"

References

Azzalini, A. (1985). A class of distributions which includes the normal ones. *Scandinavian Journal of statistics*, 12, 171-178. Azzalini, A. (1986). Further results on a class of distributions which includes the normal ones. *Statistica,* 46, 199-208.

Bali, T.G. (2007). Modelling the dynamics of interest rate volatility with the skewed fat-tailed distribution. *Annals of Operation Research,* 151(1), 151-178.

Bollerslev, T. (1986). Generalized Autoregressive Conditionally Heteroskedasticity. *Journal of Econometrics*, 31, 307–327.

Bollerslev, T. (1987). A Conditionally Heteroskedastic Time Series Model for Speculative Prices and Rates of Return. *Review of Economics and Statistics*, 69(3), 542–547.

Engle, R .F. (1982). Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of U. K. Inflation. E*conometrica,* 50(4)*, 987-1008.*

Fermandez, C., and Steel, M. F. J. (1998). On Bayesian modeling of fat tails and skewness. *Journal of the American Statistical Association*, 93, 359-369.

Gauss, C. F. (1809). Theoria motvs corporvm coelestivm in sectionibvs conicis Solem ambientivm (in latin). [10] Azzalini, A. (2005). The skew-normal distribution and related multivariate families. *Scandinavian Journal of statistics*, 32,159-188.

Hansen, B. E. (1994). Autoregressive Conditional Density Estimation. *International Economic Review, 35, 705-730.*

Jones, M. C. (2001). A skewed student-t distribution, in country-region C. A. Charalambides, M. V. Kourtras, place N. Balarishnan, eds. Probability and Statistical Models with Applications, pp. 269-278.

Jones, M. C., and Faddy, M. J. (2003). A skew extension of the t distribution, with applications. Journal of the Royal Statistical Society, 65, 159-174.

Nelson, D. (1991). Conditional Heteroskedasticity in Asset Returns: A New Approach. *Econometrica*, 59, 349-370.

Normadiah, M. R. and Yap, B. W. (2011). Power comparisons of Shapiro-Wilk, Kolmogorov-Smirnov, Lilliefors and Anderson-Darling tests. Journal of Statistical Modelling and Analytics 2(1): 21-33.

O'Hagan, A., and Leonard, T. (1976). Bayes estimation subject to uncertainty about parameter constraints, *Biometrika*, 63, 201-2012.

Olayemi, M. S., and Olubiyi, A. O. (2022). Standardized exponentiated Gumbel error innovation distribution in modelling volatility models. J.Math.Comput.Sci, 12:42. ISSN: 1927-5307

Shamiri, A., and Isa, Z. (2009). Modelling and Forecasting Volatility of the Malaysian Stock Market. *Journal of Mathematics and Statistics*, 5, 234-240.

Su, C. (2010). *Application of the EGARCH model to estimate the financial volatility of daily returns: The empirical case of China* (Master dissertation). University of Gothenburg.

Theodossiou, P. (1998) Financial data and the skewed generalized t distribution. *Management Science*, 44, 1650-1661.

William, S. G. (1908). The probable error of a mean. *Biometrika*, 6(1), 1-25.